# The Wave Nature of Electrons

Juan Gonzalez De Mendoza
Physics Department, University of Texas at Austin

April 2, 2019

**Abstract**

Observing massive particles exert wave-like behavior has changed our perspective on the nature elementary particles, and has brought us many philosophical questions as we try to understand further our universe. We measured the angle at which electrons formed constructive interference on a fluorescent screen after being diffracted by carbon crystals. Through Bragg's Law we calculated the spacing between the diffracting atoms to be $.37 \pm .08nm$, this agrees with the accepted distance between carbon crystal planes of .34nm within error bounds, verifying the wave nature of electrons just as predicted by De Broglie.

## 1 Introduction

Wave-particle duality is one of the most fundamental concepts of quantum mechanics, which is one of the biggest fields in physics today. Classical theory describes electrons as point particles. This contradicts the wave behavior of electrons, which has been experimentally observed. The wave nature of matter in general has been extremely helpful in fields such as biology, due to the very short wavelengths of electrons. They are used to reflect off of biological structures which are so small, that producing a photon with small enough wavelength to reflect off the structure would require too much energy. This brought us the invention of the electron microscope, and many more tools that take advantage of the wave behaviour of matter[1].

In the late 1800's, J. C. Maxwell calculated the speed of electromagnetic radiation, which was equal to the measured speed of light through experiments, confirming that light was an electromagnetic wave.[2] At the end of the 19th century, Einstein was able to explain the photoelectric phenomenon by describing light as a discrete flow of particles with each having a corresponding momentum[3]. This contradiction is accepted as "wave-particle duality", where light behaves as a particle or as a wave depending on the experiment.[4] In 1924, De Broglie in his PhD thesis proposed that all massive particles, which were accepted as point particles, such as electrons, could also behave as waves. He rearranged Einstein's momentum of light equation, to obtain a relationship between the momentum of a particle and its corresponding wavelength [5]. In 1925, Davisson

& Germer began performing experiments to study the reflection of electrons off ordinary nickel. After an accident in their experiment, they had to expose their nickel target to high temperatures. Following this procedure, they noticed the reflection pattern of the electron had completely changed. They were observing a constructive interference wave pattern. Later on, it was observed that the heating of the nickel target caused its atoms to rearrange into a crystal lattice, which caused electrons to diffract as waves, just as De Broglie theorized in his thesis. This was the first observation of the wave-nature of massive particles. [6]

In this experiment, we used an electron beam aimed at a carbon and nickel target. The electrons then got diffracted on this target and collided with a fluorescent screen that allowed us to visually study the diffraction pattern. We then plotted the angle of the outgoing electrons against their momentum, and from the slope we calculated the distance between two graphite layers.

## 2 Theoretical Background

As mentioned above, every massive particle has a corresponding wavelength according to De Broglie. Its wavelength is defined by

$$\lambda = \frac{h}{p}, \tag{1}$$

where $\lambda$ is the wavelength, p is the momentum of the particle and h is Plancks constant.

The equation defining constructive interference patterns after diffracting a photon through a crystal gratin is called Bragg's law and it relates the spacing between the crystal atoms and the wavelength of the photon as follows

$$2dsin\theta = n\lambda.[7] \tag{2}$$

Where d is the spacing between the atoms, theta is the incident angle of the photon and lambda its wavelength. Since we are considering electrons to behave as waves, Bragg's Law should work just as it does with photons. This relationship can be derived geometrically as depicted in Figure 1

For the purposes of our experiment, it is useful to substitute $\lambda$ with equation 1 and solve for $sin\theta$. After doing this we obtain

$$sin\theta = \frac{nh}{2p} \cdot \frac{1}{d} \tag{3}$$

From special relativity, we obtain the equation of momentum of a particle as

$$p = \gamma mv, \tag{4}$$

where v is the velocity of the particle, m is its mass and $\gamma$ is the lorentz factor equal to $\frac{1}{\sqrt{1-\frac{v}{c}^2}}$.[7]

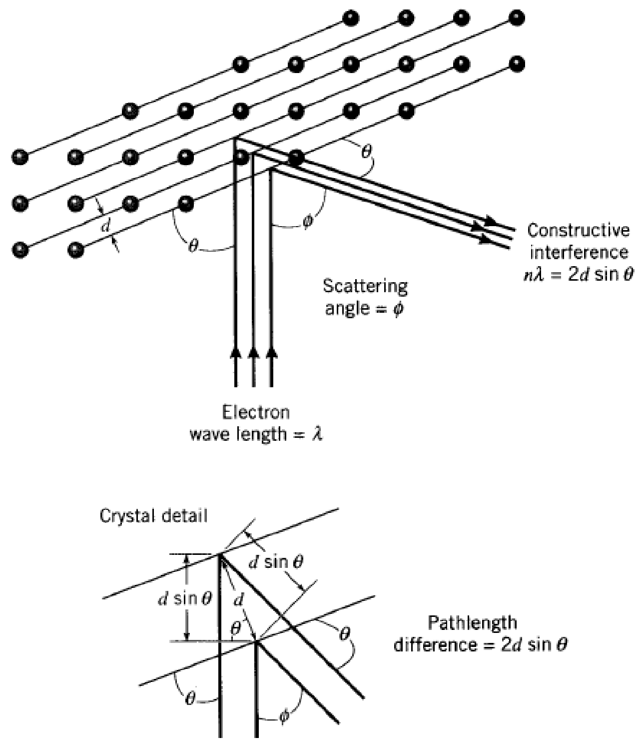


Figure 1: The incoming electron sees planes of atoms separated by a distance d. Constructive interference in path lengths occurs when the difference in path lengths from scattering by successive planes is equal to an integer times the electron wavelength. This gives the Bragg condition.[7]

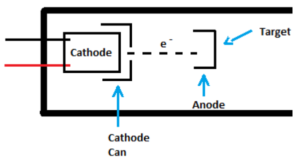


Figure 2: Apparatus used to create a collimated electron beam by surrounding the cathode with a metal shield

Our experimental setup lets us set the accelerating voltage on the electron's path. This will equal the kinetic energy that the electron will have after it has traversed it. We can calculate the electron's velocity through the following equation

$$v = c\sqrt{1 - (\frac{mc^2}{E_k + mc^2})^2}. \quad (5)$$

Where m is the mass of an electron, c is the speed of light, and $E_k$ is the Kinetic Energy of the electron, which as mentioned above, is equal to the accelerating voltage. This is because the unit of electron volts is defined as the energy an electron obtains after being accelerated through a potential difference.

# 3 Experimental Procedure

Our setup consisted of an oxide coated cathode which emitted electrons through thermionic emission. Surrounding the cathode we had a metal shield called the cathode can as depicted in Figure 2, which helped control the anode current and create a collimated electron beam. The electrons ejected would travel to the anode with an energy equal to the voltage difference between these two plates, connected and controlled by a Teltron Limited London England power supply. This electric potential was varied between 2.5 - 5 kV in increments of .5kV. The electrons would then collide with the target which was composed by a thin film of carbon deposited on a micro mesh nickel grid. Due to the inconsistency of the grid, which is composed of many tiny crystals in random orientations, many different patterns may emerge. At the right end of our setup, there was a fluorescent screen, which would let us observe visually where the electrons landed after being diffracted by the target.

As we varied the accelerating voltages, we measured the radius of the first and second rings on the fluorescent screen twice for each setting, and took the average. This was done by putting tape on the outside surface of the fluorescent screen, and marking where the rings were located, in order to be able to measure it with a caliper on a flat surface after to get more accurate measurements.

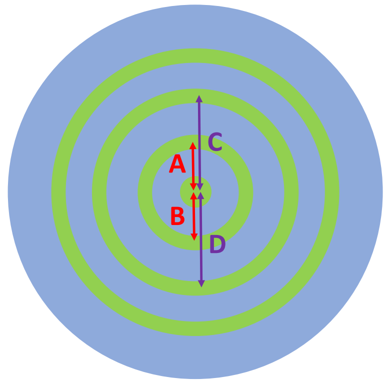


Figure 3: Pattern observed on fluorescent screen during the experiment. Radii of first two rings were measured from two sides and averaged to reduce uncertainty

# 4 Data Analysis

The pattern observed in the fluorescent screen was a wave diffraction pattern as shown in Figure 3. We measured the radius of the first ring from the top and from the bottom and took the average to reduce uncertainty. Since this pattern was projected into a curved surface, the radii measured represented some arc length on our glass sphere. To be able to calculate $\theta$ from the arc length measured some trigonometric analysis is needed as depicted in Figure 4. Through this we obtained that $\theta = arctan(\frac{sin\alpha}{1+cos\alpha})$ where $\alpha$ is the angle subtended by the radius of the ring.

By knowing the accelerating voltage, which we varied for several measurements we obtain the velocity of the electrons through equation 6, and then its momentum through equation 5. We then plotted $sin\theta$ vs. $\frac{h}{2p}$, this means that the slope of the plot represents $\frac{1}{d}$ which tells us the spacing between the atoms that diffracted the electron. If we observe Figure 5, which corresponds to the first ring n = 1, we obtained a spacing equal to $.31 \pm .13nm$. If we observe Figure 6, we see that this agrees within error bounds with the distance between two layers of the carbon crystal lattice of d = .34nm.

We then repeated the same procedure with the second ring observed, therefore setting n = 2 in Figure 7. This gave us a value of $.43 \pm .11$ which again agrees with the accepted value of .34nm within error bounds. Giving us an average of $.371 \pm .085nm$. This means that the electrons are hitting in a perpendicular direction to the carbon crystal planes.

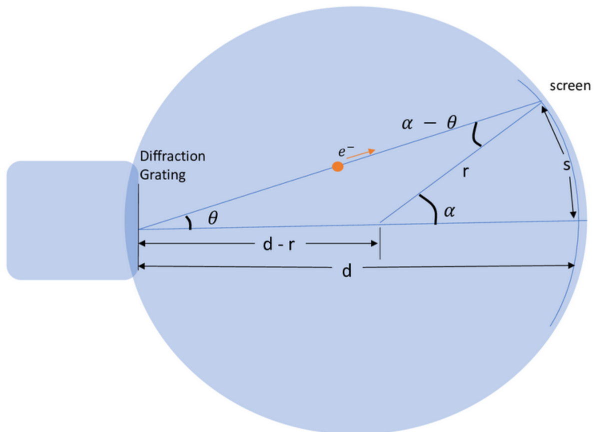


Figure 4: Trigonometric Analysis of our experiment in order to find $\theta$ and be able to use Equation 4

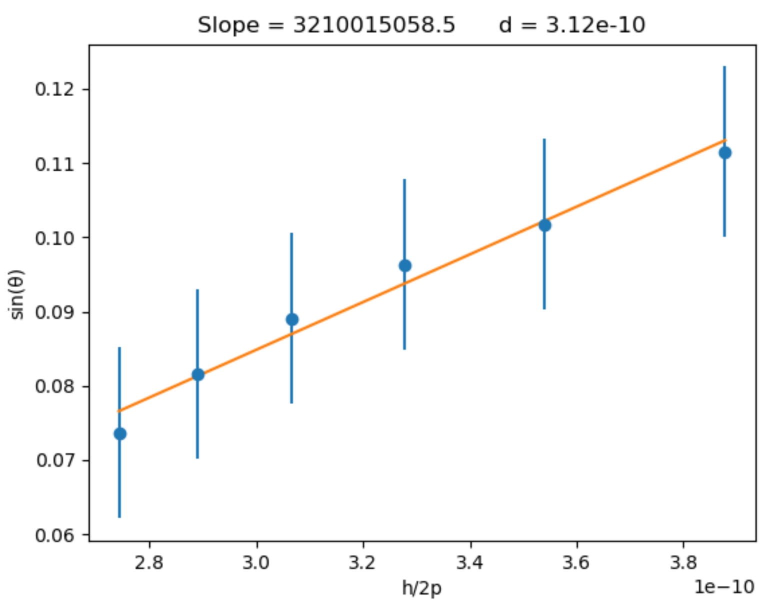


Figure 5: Plot of the sin of the angle measured on the fluorescent screen against Planck constant over twice the momentum of the electron. Giving us 1/d as the slope through a linear fit.

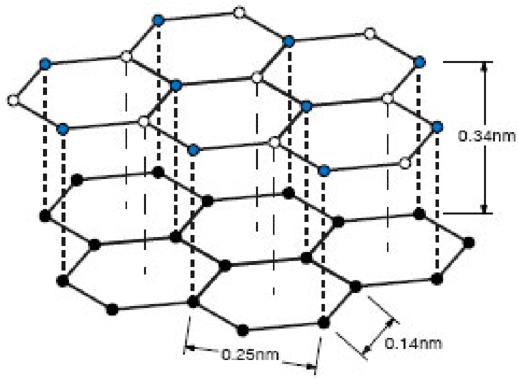


Figure 6: Accepted values for the dimensions of a carbon crystal lattice, .34nm being the distance between two different planes. The other two values are distances between atoms in the same plane

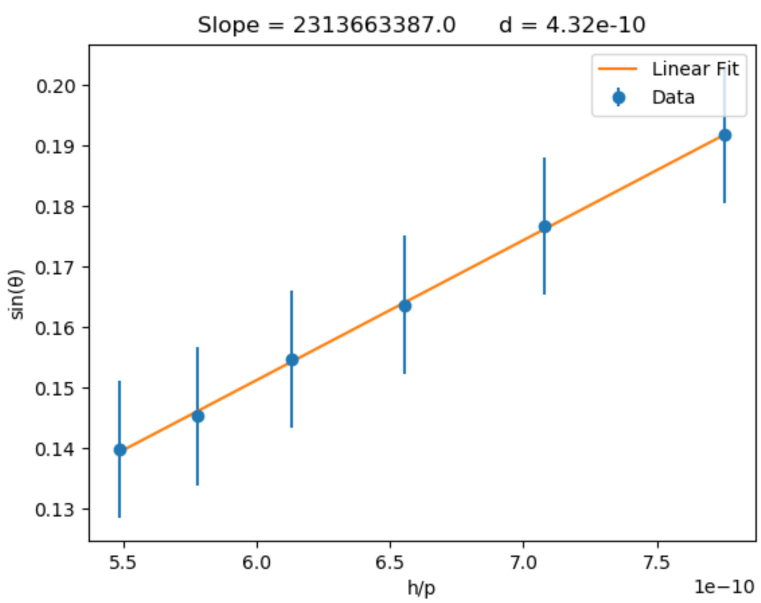


Figure 7: Plot of the sin of the angle measured on the fluorescent screen on the second ring(n = 2), against Planck constant over twice the momentum of the electron. Giving us 1/d as the slope through a linear fit.

## 4.1 Error Analysis

To measure the arc subtended by the angle $\alpha$ we placed a piece of tape on the surface of the fluorescent screen, made marks where the ring was located and its center, and took it off to measure with a caliper on a flat surface. First, we took into account the thickness of the marked lines on the tape. We measured the distances from the middle of each marking, therefore making our error for this half of the thickness of the line on the tape (averaged to $\pm 1.37mm$). We also took into account the error of the caliper $\pm .005mm$ to give us a total of $\pm 1.49mm$ for our arclength which is labeled as "s" in Figure 4. We measured the radius of the glass sphere by wrapping a string around it, and then measuring the string with a meter-stick, this yielded an error of $\pm .5mm$. Through error propagation, we calculated the corresponding error for $sin\theta$, which is dependent on each different arclength. Finally, for our uncertainty on the lattice spacing d, we obtained a covariance matrix from our linear fit, which the function that we used in python from the numpy library provided, and through more error propagation (finding the inverse of an error) we obtained our errors for d.

# 5 Conclusion

We aimed an electron beam at a carbon crystal target, which diffracted the electrons. The electrons were then absorbed by a fluorescent screen where we could visually observe a wave diffraction pattern. We measured the radii of the rings formed in this pattern and used Bragg's Law to calculate the atom spacing in a carbon crystal. We obtained a value of $.371 \pm .085nm$, this agrees with the accepted value between carbon crystal planes of .34nm within error bounds. This verifies the wave nature of matter proposed by De Broglie. This also verifies that Bragg's Law, which is typically used for electromagnetic diffraction, is not any different for massive particles.

# References


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