Measuring the Speed of Light with a Rotating Mirror

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Abstract

Knowing the speed of light with further precision improves our calculations regarding special relativity. The use of special relativity is increasing every day as we improve our GPS technology. We measured the displacement of a light beam being reflected off a rotating mirror and plotted this against the frequency of the mirror in order to use the slope to calculate the speed of light. We calculated the speed of light to be $3.032 \cdot 10^8 \pm 6.817 \cdot 10^6 m/s$, which agrees with the defined value of c = 2.99792458 $\cdot 10^8 m/s$ within error bounds, upholding similar previous experiments.

1 Introduction

The theory of special relativity changed the way physicists understand the universe. It describes phenomena that were not discovered before they were theorized, such as time dilation and length contraction. These phenomena disagree with classical theories of physics in reference frames moving close to the speed of light.[1] Special relativity has been experimentally verified, and it is important enough where everyday technology, such as the GPS system would not work if implemented under classical physics. Thus, relativity is a very important branch of physics, and it will become even more significant as we are able to reach faster speeds.

Light's velocity has been known to be finite since Olaus Roemer in 1679 performed an experiment that showed this[3]. In the 17th century, Robert Boyle proposed that there exists a medium that filled our entire universe, he called this medium "the ether". Later on, Huygens proposed that light traveled through this "ether"[2]. This implied that just like there is a preferred frame of reference for sound, which is where air molecules are at rest, there was a preferred frame for light where the ether was at rest. Michelson and Morley came along and designed an experiment to determine if the ether hypothesis was correct, and found no evidence of it. They measured the speed of light going with and against ether and found that it was always constant[4]. This

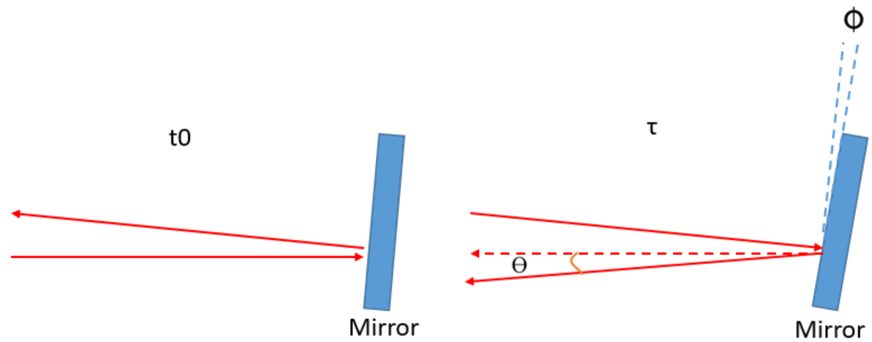

Figure 1: On the left, we can observe a light beam being reflected on a mirror, and on the right the same light beam returns after time τ through the same path but encounters that the mirror rotated through some angle ϕ, thus being reflected at a new angle θ

experiment was replicated multiple times, and it yielded the same result every time. Asides from disproving the ether, Maxwell had derived the speed of light from his equations. No matter which reference frame he used to calculate this speed, it was always constant. Physicists began accepting that the speed of light was not dependent on a reference frame, and then Einstein came along and proposed special relativity, changing physics as we knew it[5].

In our experiment, we utilized a mirror rotating at different frequencies to measure the displacement difference between a beam of light coming directly from a laser, and a second light beam reflected on said mirror. The higher the frequency, the further the mirror would rotate before the light reflected back on it, and therefore the bigger displacement we would observe on our detector. By taking several measurements, and through the slope of a graph of displacement vs. frequency we were able to calculate the speed of the light beam.

2 Theoretical Background

The time that takes a light beam to travel some distance d is $\tau = \frac{d}{c}$. In our experiment, we care about the amount of time it takes light to travel twice through the focal length of one of our lenses f, and an arbitrary distance chosen between two mirrors b, and retrace its steps back. Therefore the appropriate distance for our setup is $d = 2(f + b)$. If we substitute this distance in our time equation we obtain

$$\tau = \frac{2(f + b)}{c}, \tag{1}$$

this represents the time it takes light to get back to our rotating mirror after it is reflected from it.

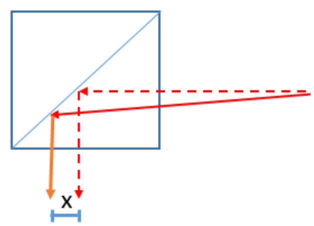

Figure 2: Two different light beams reflecting off a beam splitter separated by a distance x.

Since our mirror is rotating at some angular frequency ω, the angle change of the rotating mirror during a time interval τ is

$$\phi = \omega\tau = \frac{2\omega(f+b)}{c} \tag{2}$$

Laws of reflection state that the incident angle will be equal to the reflection angle, therefore the rotation ϕ of the mirror will cause a net change of 2ϕ compared to the first incoming beam as depicted in Figure 1. We will call this angle θ,

$$\theta = 2\phi = \frac{4\omega(f+b)}{c}. \tag{3}$$

This reflected beam is directed back at a beamsplitter, where it is compared to the original light source beam. Since it got reflected at a different θ, it will be displaced a distance x from its source beam. By looking at Figure 2, we can observe how this displacement will be equal to the arc subtended by θ, giving us

$$x = \theta a = \frac{4\omega a(f+b)}{c}, \tag{4}$$

where a is the distance between the rotating mirror and the beam splitter.

Since our equipment gave us the frequency of the mirror in Hz we substitute ω for $2\pi\nu$. After doing this we obtain

$$x = \nu\frac{8\pi a(f+b)}{c}, \tag{5}$$

where ν is the frequency of the rotation of the mirror[6]. This means that if we graph the displacement against the frequency of our mirror, our slope will be equivalent to $\frac{8\pi a(f+b)}{c}$. Thus, the speed of light can be calculated as follows

$$c = \frac{8\pi a(f+b)}{s}, \tag{6}$$

where s is the slope of a x vs. ν graph.

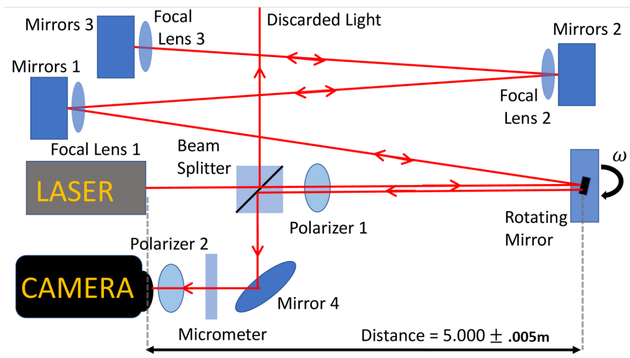

Figure 3: Experimental setup for a light beam to hit a rotating mirror, travel some distance by reflecting off mirrors, and return to the mirror after it has rotated a small amount following a slightly different return path into a camera.

Finally, since our experiment is not performed in a vacuum, light is slowed down by the index of refraction of air as follows

$$c_{air} = \frac{c_{vac}}{n_{air}}. \tag{7}$$

Since the index of refraction of air is so close to 1, it is likely this will be negligible when compared to out error. The same phenomena happens with glass, as light travels through the beam splitters and lenses, which has a more significant index of refraction, yet the distance traveled through glass is so small compared to the total path that we will mne

3 Experimental Procedure

Our setup consists of a laser aimed at a beam splitter, where some of the light is transmitted and some is reflected and discarded as depicted in Figure 3. The transmitted light would go through polarizer 1, which was oriented at 45°from the polarization of the source, thus changing the polarization of light going to the rotating mirror. Then the light beam reflects off the rotating mirror, goes through a lens which is located at focal length distance away from the rotating mirror. This way, no matter where the light beam hits the lens, it will always be able to follow its path back into the rotating mirror. After it is focused through the lens, it bounces off a combination of mirrors to travel a total of 30.000 $\pm .022m$. The light beam then reaches the rotating mirror again and due to the time it took light to travel those 30 meters, the mirror will be at a different angle, which will reflect the light beam back into the beam splitter. We then used a mirror to direct the light onto a clear micrometer used to scatter the light beam. We had a black and white camera capturing images of the micrometer where we could see the beam scattering on the micrometer. A second polarizer was placed

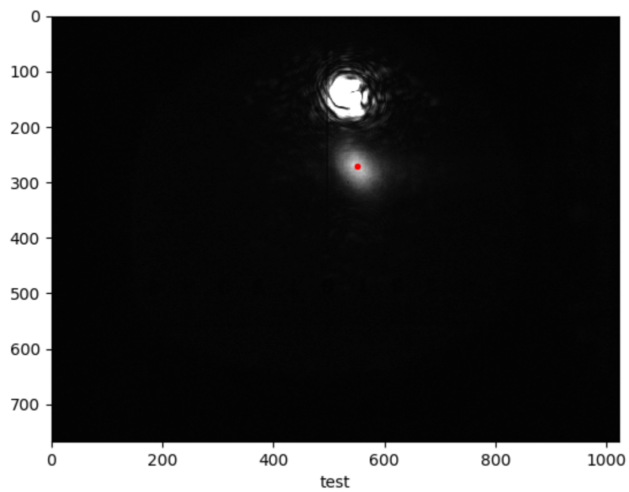

Figure 4: Picture of the micrometer. On the top we can observe an unwanted bright circle of light which was not taken into account in our analysis. Under this, we observe a dimmer light with a red dot, which represents its center of intensity found using Gaussian functions.

in front of the camera set to a 90°orientation from the laser's polarization, thus reducing interference from light sources which are not coming from the rotating mirror. We took pictures of the scattered light beam while varying the mirror's frequency, from -700 Hz to 700 Hz in 50Hz increments, all of these values have uncertainty of $\pm 5Hz$.

4 Data Analysis

In Figure 4, we can observe one of our pictures of the micrometer. On the top of the picture there is a very bright circular spot, which was a result of light reflecting off the beam splitter going directly into the camera's lens. The actual beam incoming from the rotating mirror is the dimmer circle below. This circle was around 50 pixels wide, this was due to the scattering that happens in the micrometer crystal. To get an accurate position of where the beam is actually hitting the micrometer, we fitted the intensity to a Gaussian distribution and found where the peak intensity was located. In order to not take into account the bigger circle, we only processed data for y values greater than 215 pixels, which if we look at Figure 4 was where the unwanted light ends (Note that the y pixel position increases in the downward direction). We plotted these relative displacements in pixels against their corresponding frequencies in Hz and found the slope as shown in Figure 5. The error in the pixel position

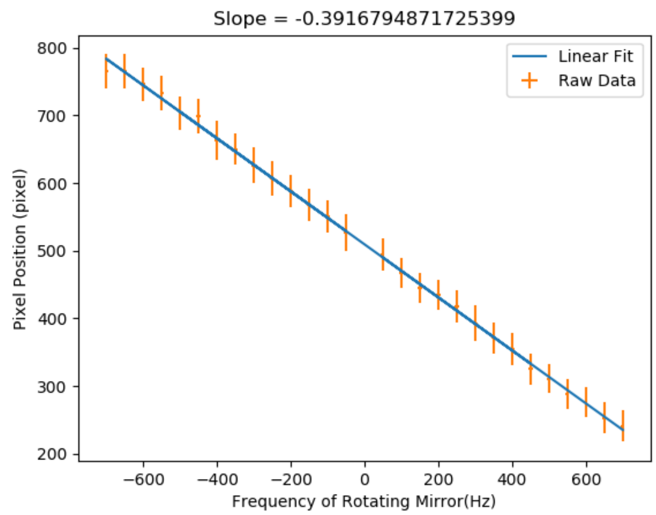

Figure 5: Plot of pixel position in the horizontal direction vs. Frequency of the rotating mirror. Slope is found in order to calculate the speed of light

comes from the uncertainty in the x_0 parameter, the peak of the distribution, for the Gaussian fit, while the error on the frequency was $\pm 5Hz$ from our rotator apparatus. In order to fit our Gaussian function and the slope, we used the curve_fit and polyfit functions from the Python Numpy library. These two return a covariance matrix with the corresponding uncertainty for our x_0 parameter and the slope.[7] This gave us a slope of -.391679 $\pm 7 \cdot 10^{-6} Pixels/Hz$, the negative is due to the arbitrary pick of measuring displacement from the 0 pixel on the left, so we can just take the absolute value of the slope. We took a picture of the micrometer where the scale was visible in order to find the appropriate Pixels/mm conversion. After some analysis using ImageJ, we found that for our experiment there are $63 \pm \sqrt{2} Pixels/mm$. Using this information we converted our slope to S.I units $s = 6.216 \cdot 10^{-6} \pm 1.3956 \cdot 10^{-7} m/Hz$. Using this slope and
a $= 5 \pm .005m$,
f $= 5 \pm .005m$,
b $= 10 \pm .01m$,
and equation 6 we obtain c $= 3.032 \cdot 10^8 \pm 6.817 \cdot 10^6 m/s$. Since the index of refraction of air is 1.0002 this yields a result that would only change decimals outside the precision our error allows. This agrees with the accepted value of c $= 2.99792458 \cdot 10^8 m/s$ within error bounds.

5 Conclusion

We measured the displacement of a light beam after it traveled a path distance of 30 meters between mirrors and reflected off a rotating mirror with different frequencies, and through the slope of a displacement vs. frequency graph, we calculated the speed of light to be c = $3.032 \cdot 10^8 \pm 6.817 \cdot 10^6 m/s$. This agrees with the defined value of c = $2.99792458 \cdot 10^8 m/s$ within error bounds. Verifying that as Roemer postulated in 1679, the speed of light is finite, and as Einstein proposed, constant.

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